Persistent spin current in a spin-orbit coupling/normal hybrid ring

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We investigate the equilibrium property of a mesoscopic ring with spin orbit (SO) interaction. It is well known that for a normal mesoscopic ring threaded by a magnetic flux, the electron acquires a Berry phase that induces the persistent (charge) current. Similarly, the spin of electron acquires a spin Berry phase traversing the ring with SO interaction. It is this spin Berry phase that induces a persistent spin current. To demonstrate its existence, we calculate the persistent spin current without accompanying charge current in the normal region in a hybrid mesoscopic ring. We point out that this persistent spin current describes the real spin motion and can be observed experimentally.

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Recently, physics of semiconductors with the spin-orbit (SO) interaction has attracted great attention, as it plays an important role for the emerging field of semiconductor spintronics.[1] SO interaction couples the spin degree of freedom of electrons to their orbital motions, thereby giving rise to a useful way to manipulate and control the electron spin by an external electric field or a gate voltage. Interesting effects resulting from SO interaction have been predicted. For example, using the effect of spin precessions due to the Rashba SO interaction Datta and Das proposed a spin-transistor more than ten years ago.[2] Very recently, a very interesting effect, the intrinsic spin Hall effect, is theoretically predicted by Murakami *et.al.* and Sinova *et.al.*,[3,4] that a substantial amount of dissipationless spin current can be generated from the interplay of the electric field and the SO coupling. Since then, the spin Hall effect has generated tremendous interests with a great deal of works focusing in the field of spintronics.

In this Letter, we explore another interesting effect that a persistent spin current without accompanying charge current exists in a coherent mesoscopic semiconductor ring with simplectic symmetry, i.e., with SO coupling but maintaining the time reversal symmetry. More than two decades ago, the persistent (charge) current in a mesoscopic ring threaded by a magnetic flux has been predicted theoretically,[5] and later observed experimentally in the early 1990s.[6] It is now well known that the persistent charge current is a pure quantum effect and can sustain without dissipation in the equilibrium case. There has also been many investigations on the persistent spin current.[7,8,9,10] For example, in a mesoscopic ring with a crown-shape inhomogeneous magnetic field[7] or threaded by a magnetic flux[8], the persistent spin current has been predicted and is related to the Berry's phase. Recently, the persistent spin current carried by Bosonic excitations has also been predicted in a Heisenberg ring with the magnetic field or in the ferromagnetic material.[9] The reason that the persistent spin current exists may be explained as follows. Due to the magnetic field or the magnetic flux, there are persistent flows of both spin up and down electrons. In the absence of SO coupling, this gives rise to the well known persistent charge current. In the presence of SO coupling or magnetic field, the persistent charge current is spin polarized resulting a nonzero persistent spin current. Hence the origin of this persistent spin current is the same as that of persistent charge current so that the persistent spin current always accompanies with a persistent charge current.

Up to now, the issue that whether the persistent spin current without accompanying charge current (a pure persistent spin current) can be induced solely by SO interaction at zero magnetic flux or magnetic field has not been addressed.[11] In the present Letter, we show that a non-magnetic semiconducting ring with SO interaction can sustain a pure persistent spin current in the absence of the external magnetic field or magnetic flux. Since the magnetic flux or magnetic field acts like a "driving force" for the persistent charge current, one naturally looks for the analogous "driving force" in the spin case. To discuss this question, let us consider two devices. The first device consists of a mesoscopic ring (without SO interaction) where a magnetic atom with a magnetic dipole moment is placed at the center of the ring (see Fig.1a). In the second device the magnetic atom is replaced by a charged atom, e.g., an ion (see Fig.1b). The magnetic atom produces a vector potential $\mathbf{A}$ on the perimeter of the ring which drives the persistent charge current. By analogy, a charged atom which produces a scalar potential ϕ on the perimeter of the same ring should drive a persistent spin current. Since the presence of this ionic center generates a SO interaction in the relativistic limit, we expect that SO interaction which plays the role of the spin "driving force" will induce a pure persistent spin current.[12]

The existence of the pure persistent spin current can be examined from another point of view using Berry phase. It is well known that an electron circulating a ring with a non-uniform magnetic field or magnetic flux acquires a

geometric phase (Berry phase).[13] It has been discovered by Loss et al that it is this Berry phase χ that induces the well known persistent charge current.[7] Assuming that the electron wavelength is much smaller than the perimeter of the ring and the electron motion is quasi-classical, let us examine an electron with spin σ traverses slowly along the ring with only SO interaction.[14] Due to the SO interaction, the spin of this electron precesses and acquires a geometric phase after the electron returns to its starting point.[14] This is the so-called spin Berry phase[14]. The spin Berry phase due to Rashba SO interaction for an electron with spin σ moving in the clockwise direction is found to be[14] $\chi_\sigma = \sigma\pi$ where $\sigma = \pm$ for $\sigma =\uparrow,\downarrow$. From the physical picture due to Loss et al,[7] the spin Berry phase χ_+ for the spin up electron induces a clockwise persistent spin polarized current I_1. Similarly, the Berry phase χ_- induces a counter-clockwise persistent spin polarized current with the polarization exactly opposite to that of I_1 since our system has time reversal symmetry. As a result, the spin Berry phase due to SO interaction will induce a pure persistent spin current.

Now we present an example to show that indeed a pure persistent spin current can exist for a semiconducting ring with SO interaction. In the presence of SO interaction, the spin of an electron experiences a torque and hence σ_i $(i = x, y, z)$ is not a good quantum number anymore. Because of this, the spin current is not conserved using the conventional definition. At present there are controversies on whether one should define a conserved spin current or whether there exists a conserved spin current.[15,16] In another word, so far there is no consensus on the definition for the spin current in the presence of SO interaction. In the present work, we do not wish to address the issue of this controversy. We avoid this controversy by considering a one-dimensional mesoscopic semiconducting ring that consists of a Rashba SO coupling region and a normal region without SO interaction as shown in Fig.1c.[12] Since there is no spin-flip in this normal region, the spin current can be defined without controversy.

The Hamiltonian of our system is given by:[8,17]

$$H = -E_a\frac{\partial^2}{\partial\varphi^2} - \frac{i\sigma_r}{2a}\left[\alpha_R(\varphi)\frac{\partial}{\partial\varphi} + \frac{\partial}{\partial\varphi}\alpha_R(\varphi)\right] - i\frac{\alpha_R(\varphi)}{2a}\sigma_\varphi \tag{1}$$

where $E_a = \hbar^2/2ma^2$, a is the radius of the ring, m is the effective mass of the electron, $\sigma_r = \sigma_x\cos\varphi + \sigma_y\sin\varphi$, and $\sigma_\varphi = -\sigma_x\sin\varphi + \sigma_y\cos\varphi$. $\alpha_R(\varphi)$ is the strength of the Rashba SO interaction, $\alpha_R(\varphi) = 0$ while $0 < \varphi < \Phi_0$, i.e. in the normal region, and $\alpha_R(\varphi)$ is a constant α_R in the SO coupling region with $\Phi_0 < \varphi < 2\pi$.

The eigenstates of Hamiltonian (1) can be solved numerically in the following way. First in the Rashba SO coupling region ($\alpha_R \neq 0$), the equation $H\Psi(\varphi) = E\Psi(\varphi)$ has four independent solutions $\Psi_i^{SO}(\varphi)$ $(i = 1, 2, 3, 4)$:[8]

$$\Psi_{1/2}^{SO}(\varphi) = \begin{pmatrix} \cos(\theta/2)e^{ik_{1/2}\varphi} \\ -\sin(\theta/2)e^{i(k_{1/2}+1)\varphi} \end{pmatrix}, \tag{2}$$

and $\Psi_{3/4}^{SO} = \hat{T}\Psi_{1/2}^{SO}$ with $\hat{T}$ being the time-reversal operator. In Eq.(2), the wave vectors $k_{1/2} = -1/2 + 1/(2\cos\theta) \pm (1/2)\sqrt{(1/\cos^2\theta) - 1 + 4E/E_a}$, and the angle θ is given by $\tan(\theta) = \alpha_R/(aE_a)$. Similarly, in the normal region ($0 < \varphi < \Phi_0$), the Schödinger equation has four independent solutions: $\Psi_1^N(\varphi) = (1,0)^\dagger e^{ik\varphi}$, $\Psi_2^N(\varphi) = (1,0)^\dagger e^{-ik\varphi}$, and $\Psi_{3/4}^N = \hat{T}\Psi_{1/2}^N$. Secondly, the eigen wave function $\Psi(\varphi)$ with the eigen energy E can be represented as:

$$\Psi(\varphi) = \begin{cases} \sum_i a_i\Psi_i^N(\varphi), & while\ \ 0 < \varphi < \Phi_0 \\ \sum_i b_i\Psi_i^{SO}(\varphi), & while\ \ \Phi_0 < \varphi < 2\pi \end{cases} \tag{3}$$

where a_i and b_i $(i = 1, 2, 3, 4)$ are constants to be determined by the boundary conditions at the interfaces $\varphi = 0$ and Φ_0. Here the boundary conditions are the continuity of the wave function $\Psi(\varphi)|_{\varphi=0^+/\Phi_0^+} = \Psi(\varphi)|_{\varphi=2\pi^-/\Phi_0^-}$ and the continuity of its flux $\hat{v}_\varphi\Psi|_{\varphi=0^+/\Phi_0^+} = \hat{v}_\varphi\Psi|_{\varphi=2\pi^-/\Phi_0^-}$, where $\hat{v}_\varphi = a(\partial\varphi/\partial t) \sim \partial/\partial\varphi + (i/2)\sigma_r\tan(\theta)$ is the velocity operator. By using the boundary conditions, we obtain eight series of linear equations with the variables $\{a_i, b_i\}$. Then, by setting the determinant of the coefficients to be zero, the eigenvalues E_n are obtained numerically. Fig.1d shows E_n versus the Rashba SO strength α_R. For the normal ring ($\alpha_R = 0$), the eigenvalues are n^2E_a with fourfold degeneracy, and the corresponding eigenstates are $(1,0)^\dagger e^{\pm in\varphi}$ and $(0,1)^\dagger e^{\pm in\varphi}$. As the SO interaction is turned on the degenerate energy levels split while maintaining twofold Kramers degeneracy. The higher the energy level, the larger this energy split. Typically, the splits are on the order of E_a at $\alpha_R = 10^{-11}eVm$, with $E_a \approx 0.42meV$ for the ring's radius $a = 50nm$ and the effective mass $m = 0.036m_e$. The eigenvalues E_n versus the normal region's angle Φ_0 also are shown (see Fig.1f). For $\Phi_0 = 2\pi$, the whole ring is normal and E_n are fourfold degenerate. When Φ_0 is away from 2π, the degeneracy are split into two, and the splits are larger with the smaller Φ_0. When $\Phi_0 = 0$, the whole ring has the Rashba SO interaction, and the split reaches the maximum.

Since E_n is twofold degenerated, we obtain two eigenstates from Eqs.(4-10) for each E_n, which are labeled $\Psi_n(\varphi)$ and $\hat{T}\Psi_n(\varphi)$. With the wave functions, the spin current contributed from the level n can be calculated straightforwardly by using $I_{Si}^n(\varphi) = Re\Psi_n^\dagger\hat{v}_\varphi\hat{\sigma}_i\Psi_n$ $(i = x, y, z)$. Since there is a controversy about the definition of spin current in the SO region, we will calculate the spin current only in the normal region. We note that the spin current is conserved in the normal region independent of the angle coordinate φ. Fig.2 shows the spin current I_{Si}^n versus the Rashba SO strength α_R for $\Phi_0 = \pi$ (i.e. a half of the ring is normal and another half has the SO interaction). Our results show that I_{Sx}^n is exactly zero for all level n, and I_{Sy}^n and I_{Sz}^n exhibit the oscillatory pattern with α_R. A $\pi/2$-phase shift between I_{Sy}^n and I_{Sz}^n is observed with $\sqrt{(I_{Sy}^n)^2 + (I_{Sz}^n)^2}$ approximately constant.

For two adjacent levels $2n-1$ and $2n$, their spin current have opposite signs, and $I_{Si}^{2n-1}+I_{Si}^{2n}=0$ if $\alpha_R=0$. We note that the spin current I_{Si}^n is quite large. For example, the value E_a is equivalent to the spin current of a moving electron in the ring with its speed $4\times 10^5 m/s$.

Now we calculate the equilibrium total spin current I_{Si} contributed from all occupied energy levels: $I_{Si} = 2\sum_n I_{Si}^n f(E_n)$, where $f(E_n)=1/\{\exp[(E_n-E_f)/k_BT]+1\}$ with the Fermi energy E_f and the temperature T. The factor 2 is due to the Kramers degeneracy. The persistent charge current and the equilibrium spin accumulation are found to be zero because the system has the time-reversal symmetry and the states Ψ_n and $\hat{T}\Psi_n$ have completely opposite charge current and the spin accumulation. Fig.3a,b show the total spin currents I_{Si} versus the Rashba SO strength α_R for different E_f. One of the main results is that the spin current indeed is non-zero when $\alpha_R\neq 0$. The persistent spin currents I_{Si} in Fig.3 have the following features. At $\alpha_R=0$ the whole ring is normal, so I_{Si} is exactly zero. With α_R increasing, I_{Si} first increases and it then oscillates for the large α_R. At certain α_R, there is a jump in the curve of I_{Si} versus α_R. Because for this α_R the Fermi energy E_F is in line with a level E_n, leading to a change of its occupation. At zero temperature, the jump is abrupt as shown in Fig.3a,b. But at finite temperature, this jump will be slightly smooth. In fact, these results are similar with the persistent (charge) current in the mesoscopic ring.[5]

The spin current I_{Si} versus the angle of normal region Φ_0 at a fixed $\alpha_R = 3\times 10^{-11} eVm$ is shown in Fig.3c. While $\Phi_0=2\pi$, the whole ring is normal, and $I_{Sx/y/z}=0$. When Φ_0 is away from 2π, the spin current $I_{Sx/y/z}$ emerge. There perhaps exist the jump in the curve I_{Si}-Φ_0 which behaviors is similar as the jump in the curve I_{Si}-α_R. In particular, while Φ_0 tends to zero, i.e. the normal region tends vanishing, the spin current I_{Sx} and I_{Sz} still exist. This means that the normal region is not necessary for generating I_{Si}.

From our results several observations are in order: (i) Since the spin current is calculated in the normal region with no spin-flip, the spin current is conserved. Hence the existence of spin current in the present system is not due to the definition of the spin current. We consider that this spin current describes the real motion of spins. In addition, the spin current should also exist in the SO coupling region. (ii) The present system is non-magnetic so this spin current is solely due to the SO interaction. (iii) This spin current calculated from the eigenstate of the system is an equilibrium property of the system. It can sustain without dissipation in the equilibrium case, i.e. it is a persistent spin current. (iv) Besides the ring, the device can also be of other shapes, e.g. a disc device or a quasi one-dimension quantum wire, and so on. So it is a generic feature that a pure persistent spin current appears in the SO coupling semiconductor as long as the size of the device is within the phase coherence length. (v) It is well known that the persistent charge current can generate a magnetic field. Similarly, the persistent spin current can generate an electric field, which offers a way of detecting it.[9,18]

In summary, we show that a persistent spin current without accompanying charge current exists in a semiconducting ring with only spin-orbit (SO) interaction so that the time reversal symmetry is retained. It is the spin Berry phase due to the SO interaction that induces the pure persistent spin current in the ring. We demonstrate the existence of the persistent spin current in normal region of a SO coupling/normal hybrid device. We point out that this spin current describes the real spin motion and can be measured experimentally. Our persistent spin current in a semiconducting ring with SO interaction is an analog of the persistent charge current in the mesoscopic ring threaded by magnetic flux.

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[12] Consider the case that there is a point charge at the center of the ring that gives rise to a Thomas SO interaction. We

find that the persistent spin current also exists, similar as for the Rashba SO interaction case discussed in the text.

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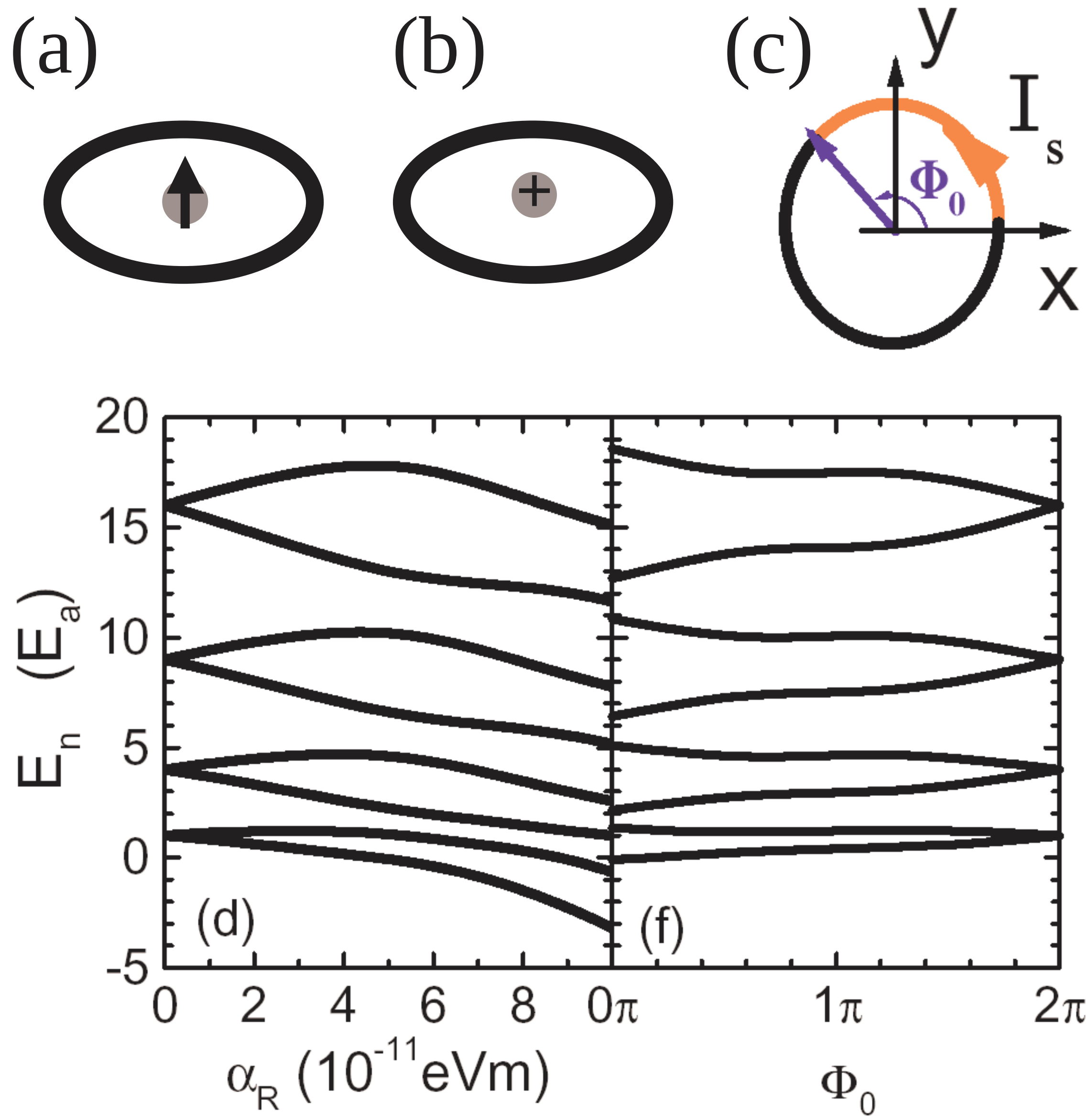
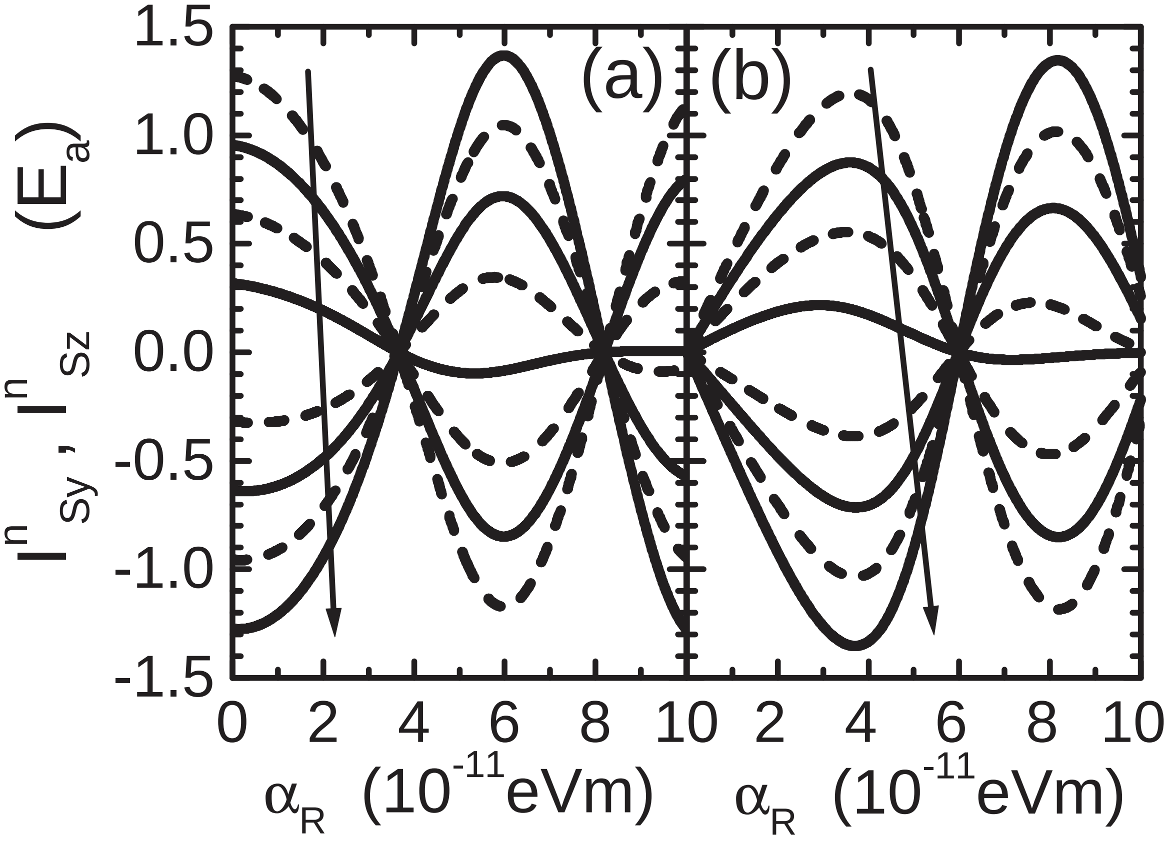
FIG. 1: (Color online) (a) and (b) are the schematic diagrams for a mesoscopic ring with a magnetic atom or an ion at its center. (c) Schematic diagram for a hybrid mesoscopic ring having Rashba SO interaction in part of the ring and the other part being normal. (d) and (f) show the eigen energies E_n vs. α_R for $\Phi_0 = \pi$ and vs. Φ_0 for $\alpha_R = 3 \times 10^{-11} eVm$, respectively. The ring's radius $a = 50nm$.

FIG. 2: I^n_{Sy} (a) and I^n_{Sz} (b) vs α_R for $\Phi_0 = \pi$ and $a = 50nm$. Along the arrow direction, $n = 7$, 5, 3, 1, 2, 4, 6, and 8.

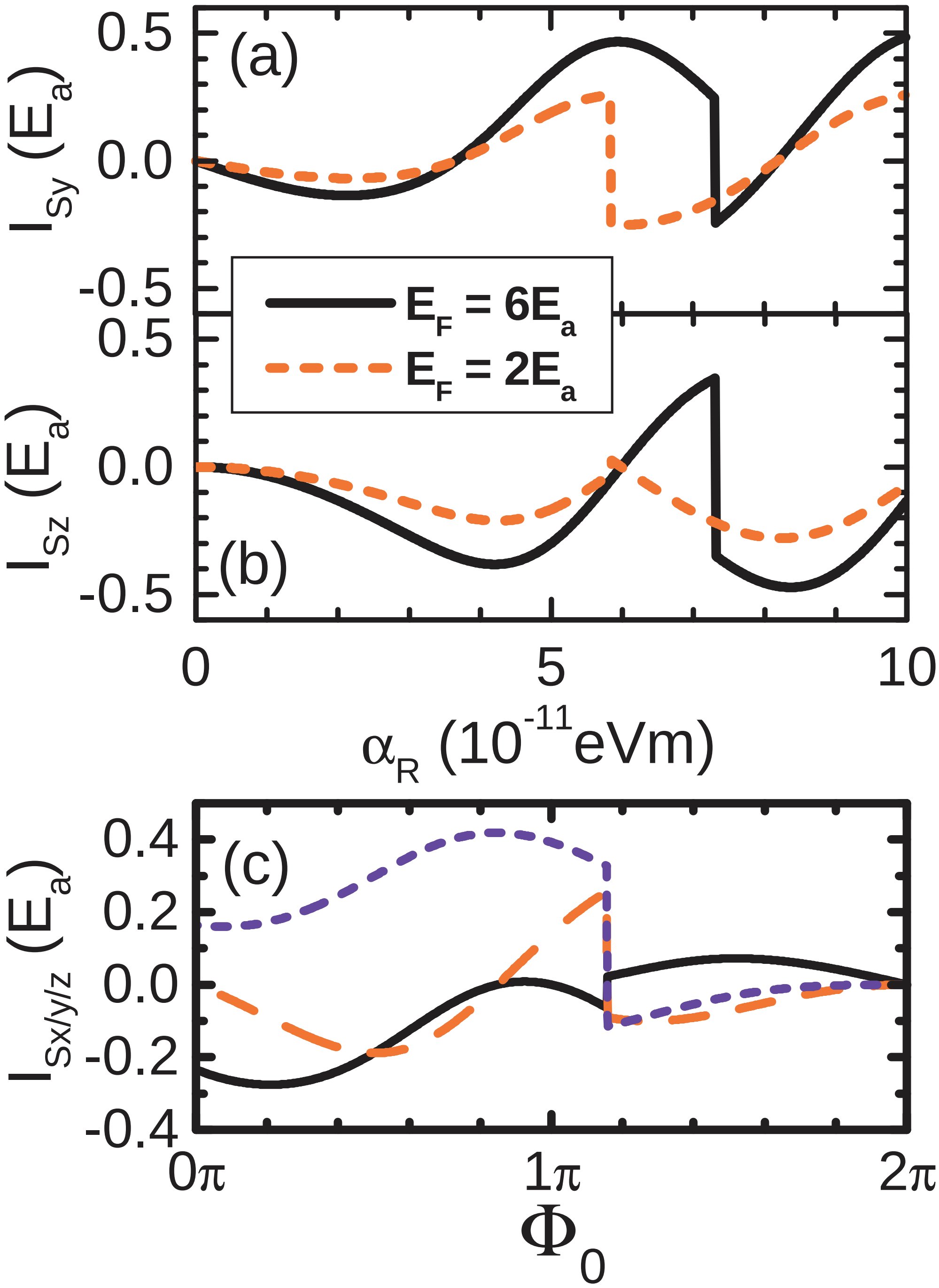
FIG. 3: (Color online) (a) and (b) show I_{Sy} and I_{Sz} vs α_R for $\Phi_0 = \pi$. (c) shows $I_{Sx/y/z}$ vs the normal region's angle Φ_0 for $\alpha_R = 3 \times 10^{-11} eVm$ and $E_f = 3E_a$. The solid, dashed, and dotted lines correspond to I_{Sx}, I_{Sy}, and I_{Sz}, respectively. The ring's radius $a = 50nm$ and the temperature $T = 0$.



(a)
(b)
(c)
y
x
I_s
Φ_0
20
15
10
5
0
-5
E_n (E_a)
(d)
(f)
0
2
4
6
8
0π
1π
2π
α_R (10^{-11}eVm)
Φ_0



(a)
(b)
I^n_{Sy} , I^n_{Sz} (E_a)
α_R (10^{-11}eVm)
α_R (10^{-11}eVm)
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
0 2 4 6 8 10
0 2 4 6 8 10



(a)
(b)
(c)
0.5
0.0
-0.5
0.4
0.2
0.0
-0.2
-0.4
0
5
10
0π
1π
2π
I_{Sy} (E_a)
I_{Sz} (E_a)
$I_{Sx/y/z}$ (E_a)
α_R (10^{-11}eVm)
Φ_0
$E_F = 6E_a$
$E_F = 2E_a$